

03_RandomEffects

QuantFit Estimator Standard Operating Procedure

SOP: Random Effects (RE / GLS)

Panel estimator that treats entity heterogeneity as a random component

=> Use RE when entity effects are uncorrelated with the regressors - confirmed by Hausman.

1. Purpose

RE assumes entity heterogeneity α_i is a random draw from a population, uncorrelated with X. The estimator is feasible GLS: a weighted combination of within and between variation, more efficient than FE when its assumptions hold.

2. When to use this estimator

Panel data where entity heterogeneity is plausibly random (sample drawn from a larger population).

Hausman test fails to reject $H_0: \text{cov}(\alpha_i, X_{it}) = 0$.

Need to estimate effects of time-invariant regressors (gender, sector, etc.).

Small T relative to N where FE is inefficient.

3. Required data structure

Panel data with entity ID column.

Numeric Y and X; time-invariant regressors are fine (RE retains them).

Adequate within and between variation in regressors.

4. Mathematical formulation

Quasi-demeaning with weight θ derived from variance components:

$$Y_{it} - \theta \alpha_i = (X_{it} - \theta X_{i0})' \beta + \nu_{it}$$

$$\theta = 1 - \sqrt{\sigma^2_{\epsilon} / (T \sigma^2_{\alpha} + \sigma^2_{\epsilon})}$$

$\beta_{RE} = \text{OLS on the quasi-demeaned data}$

5. Pre-estimation diagnostics

Stationarity per variable (panel unit root tests).

Compute Hausman test ($H_p < 0.05 \Rightarrow$ FE preferred over RE).

Breusch-Pagan LM test ($H_0: \sigma^2_{\alpha} = 0$; rejection \Rightarrow RE preferred over pooled OLS).

VIF < 5 across regressors.

6. Estimation procedure

Run pooled OLS to obtain σ^2_{ϵ} .

Run between-entity regression on entity means to obtain σ^2_b .

Solve for $\sigma^2_{\alpha} = \sigma^2_b - \sigma^2_{\epsilon} / T$.

Compute θ and quasi-demean Y and X .

Run OLS on quasi-demeaned data; SE come from the GLS variance formula.

7. Output produced

8. Output interpretation

β_j is interpreted similarly to OLS: average effect on Y per unit X .

Hausman $p \geq 0.05$: RE is consistent and more efficient than FE.

Hausman $p < 0.05$: RE is inconsistent; switch to FE.

BP LM rejects: heterogeneity exists; pooled OLS is inefficient.

9. Post-estimation diagnostics

Hausman test result must accompany the RE estimate.

Examine residual autocorrelation and clustering.

Robust SE: clustered by entity for valid inference.

10. Common pitfalls

RE is biased and inconsistent when α_i is correlated with X - always run Hausman.

Reporting RE without Hausman is a red flag for a serious reviewer.

Time-invariant regressors are kept but may absorb part of α_i ; interpret with care.

11. Reporting checklist

Hausman test statistic, df, p-value alongside coefficients.

Breusch-Pagan LM result.

Within / between / overall R^2 for comparison with FE.

Cluster-robust SE.

12. References

Wooldridge (2010). *Econometric Analysis of Cross Section and Panel Data*, 2nd ed.

Hausman, J. A. (1978). Specification Tests in Econometrics. *Econometrica*.

Breusch, T. S., Pagan, A. R. (1980). The Lagrange Multiplier Test. *Review of Economic Studies*.

Field | Meaning

coefficients | GLS slopes β_j

metadata['hausmanH'] | Hausman χ^2 statistic
metadata['hausmanP'] | Hausman p-value (small => prefer FE)
metadata['bpLM'] | Breusch-Pagan LM statistic
metadata['bpP'] | BP p-value (small => prefer RE over OLS)
metadata['theta'] | Quasi-demeaning weight
residuals / fitted | Standard residuals and fitted values