15_VECM

QuantFit Estimator Standard Operating Procedure

SOP: Vector Error Correction Model (VECM)

VAR for cointegrated I(1) systems

=> Use VECM when k>=2 I(1) variables are cointegrated - the levels VAR is non-stationary, the VECM is.

1. Purpose

VECM expresses a cointegrated VAR in error-correction form. The Johansen procedure identifies the cointegrating rank r ? [1, k-1] and estimates the cointegrating vectors beta and adjustment matrix alpha. Short-run dynamics are captured by lagged Delta-terms.

2. When to use this estimator

All variables I(1) with at least one cointegrating relationship (Johansen rank >= 1).

Multivariate framework where VAR in levels would be misspecified.

3. Required data structure

All k series I(1) per Stage 3.

Johansen test confirms r >= 1 cointegrating vectors.

T ? 2 x k x p typical.

4. Mathematical formulation

DeltaY_t = alphabeta' Y_{t-1} + Sum Gamma_i DeltaY_{t-i} + epsilon_t

beta: k x r matrix of cointegrating vectors (long-run equilibria).

alpha: k x r adjustment matrix; (alphabeta') is rank-r.

Gamma_i: short-run k x k coefficient matrices on lagged differences.

Reduced-rank regression solves alpha and beta simultaneously (Johansen).

5. Pre-estimation diagnostics

All variables I(1).

Johansen trace and max-eigenvalue tests for rank r.

Lag selection in the underlying VAR (AIC / BIC / HQ).

6. Estimation procedure

Determine VAR lag order p in levels.

Run Johansen reduced-rank regression to get r.

Estimate DeltaY_t = alphabeta' Y_{t-1} + Sum Gamma DeltaY_{t-i} + epsilon_t

with rank r.

Reconstruct levels VAR for Cholesky IRF / FEVD.

Run residual diagnostics (autocorrelation, normality, stability).

7. Output produced

8. Output interpretation

beta'Y_{t-1} are the long-run equilibrium errors; alpha is the adjustment toward them.

Sign and magnitude of alpha reveals which variables bear the burden of correction.

IRF / FEVD interpreted same as VAR after the levels reconstruction.

9. Post-estimation diagnostics

Stability (spectral radius < 1 after rank reconstruction).

Residual autocorrelation, normality, ARCH-LM.

Robustness across rank choice.

10. Common pitfalls

Mis-specifying the rank produces inconsistent alpha and beta.

Mixing I(0) and I(1) variables - drop I(0) ones or transform first.

Cholesky ordering still matters for IRF interpretation post-reconstruction.

11. Reporting checklist

Johansen trace / max-eig test results and chosen rank.

Cointegrating vectors beta (normalised).

Adjustment matrix alpha with significance.

IRF / FEVD panels.

Stability and residual diagnostics.

12. References

Johansen, S. (1995). Likelihood-Based Inference in Cointegrated Vector Autoregressive Models. Oxford.

Lütkepohl, H. (2005). New Introduction to Multiple Time Series Analysis. Springer.

Field | Meaning

metadata['cointegratingRank'] | Johansen-selected r

metadata['cointegratingVectors'] | beta as flattened text block

metadata['adjustmentMatrix'] | alpha

varIRF / varFEVD | Computed from the implied levels VAR

companionSpectralRadius | Stability

residuals | VECM residuals